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Ring and Discs

Moment of Inertia Demonstration Kit

Introduction

A ring and a disc begin to roll down an inclined plane at the same time. Which one will win the race? Or, will they reach the bottom at the same time? Let's find out.

Concepts

- Rotational motion
- Potential energy
- Moment of inertia
- Kinetic energy
- Newton's Laws of Motion

Materials

Disc, solid, 3.5″ diameter*

Disc, solid, 5″ diameter*

Ring, 3.5″ diameter*

**Materials included in kit*

Balance, 1-gram precision

Inclined plane or wood board, 1-m or longer

Ruler or meter stick

Wood block or book

Safety Precautions

Although the materials in this kit are considered nonhazardous, please follow proper laboratory safety guidelines.

Preparation

Prepare an inclined plane by elevating one end of a commercial inclined plane, or a thin wood board (about one meter long), with a block of wood or a textbook. Raise to a height of 5–10 centimeters from the floor or table top. Make sure the inclined plane or board is level, and there is no sideways tilt.

Procedure

1. Measure the mass of the 3.5″ diameter ring using a balance, and note its mass. Repeat for the 3.5″ diameter solid disc. Both should have approximately the same mass. Report this result to the students.
2. Place a ruler flat at the top of the inclined plane. Make sure it does not slide down. Position the 3.5″ ring and 3.5″ solid disc behind the ruler so that they will roll straight down the inclined plane (see Figure 1). Adjust the ruler, if necessary, so that the centers of the ring and disc are at the same starting position on the inclined plane (i.e., the points on the ring and disc in contact with the inclined plane are at the same height).

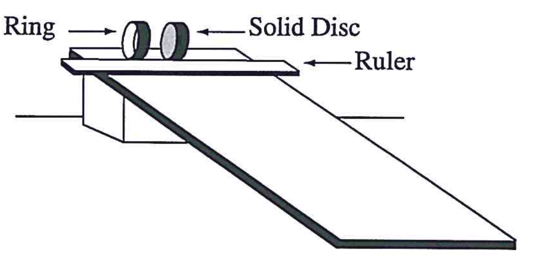


Figure 1.

3. Have the students make a prediction about which object will reach the bottom of the inclined plane first. Ask them to support their claim. (Most students will predict that the ring will reach the bottom first.)
4. Quickly remove the ruler or meter stick from the base of the ring and disc so they both begin to roll down the inclined plane at the same time. Observe them roll down the plane.

5. Discuss the results. (The solid disc will reach the bottom of the incline plane first.)
6. Next, follow procedure 1–5 using the 3.5″ diameter ring, 3.5″ diameter solid disc and the 5″ diameter solid disc. Race all three at the same time. (The 5″ diameter solid disc is approximately twice the mass of the 3.5″ diameter solid disc.)
7. Again, discuss the results. Most students will predict that the larger solid disc will be the slowest. However, both solid discs should reach the bottom at the same time, followed by the ring. This further emphasizes Newton's law that motion under constant acceleration (without friction) is independent of mass or size. However, for rolling objects, the shape *is* important—whether the shape of the object is a hollow sphere, a solid ball, a tube, or a solid disc—but the mass or the diameter are not.

Disposal

All materials may be saved for future use.

Tips

- This demonstration introduces the concept of the moment of inertia, and reinforces the elementary principles of mechanics and Newton's laws of motion.
- Use objects with different shapes, such as hollow spheres, balls or solid tubes, to further enhance these ideas. For best results, use objects that have smooth edges and are relatively massive so that air resistance is negligible.
- Alternative to propping up one end of the inclined plane or wood board with a block of wood or a book: Lay the inclined plane or wood board flat on the tabletop. Position the ring and discs near one of the ends of the inclined plane or board so that they are at rest and their contact points with the inclined plane are the same distance away from the end. To start the objects rolling along the inclined plane at the same time, slightly lift the end of the inclined plane where the ring and disc are positioned. The end only needs to be lifted a few centimeters and it should be held at this height until the race is finished.
- Do not raise the inclined plane up to a large angle because this may cause the ring and disc to slip down the inclined plane rather than roll, which would skew the "expected" results for the rolling objects.
- A towel or other soft barricade can be placed at the end of the inclined plane to stop the ring and discs once they finish the race.
- The moment of inertia differences (speed of descent differences) can best be observed when using the longest inclined plane available. Inclined planes one meter long or longer will work best.

Discussion

Why does a solid disc roll down an inclined plane faster than a ring? The solid disc beats the ring because it has a lower "resistance" to motion. All mass has the property of resisting a change in motion, or *inertia*. An object in motion wants to stay in motion, and an object at rest wants to stay at rest. For linear motion, the "resistance" is based on the mass of the object (it is harder to start or stop a train than it is to start or stop a car). For rotational motion (spinning motion), the "resistance" is a property based on the mass *and* the spatial distribution of the mass around a point of rotation (or *axis of rotation*). This specialized case of inertia is called *moment of inertia* (or, sometimes, *rotational inertia*). The distribution of the mass affects the moment of inertia in such a way that the further the bulk of the mass is distributed from the point of rotation, the larger the moment of inertia will be, and therefore, the harder it will be to change the object's motion. In the *Ring and Discs* demonstration, the 3.5″ ring and the 3.5″ solid disc have similar mass, but the 3.5″ ring has a larger moment of inertia than the 3.5″ solid disc because all the mass is distributed at the edge, far away from the center of the ring (the axis of rotation for the rolling ring). The mass in the solid disc is spread out evenly throughout the entire disc and therefore the "bulk" of the mass is located closer to center of the disc (the axis of rotation for the rolling disc), so the moment of inertia is lower. The object with the lower moment of inertia will move faster and win. When the ring and disc start to roll down the inclined plane, because of the force of gravity, the ring "resists" the force of gravity more than the solid disc, meaning it moves slower and finishes the race after the solid disc.

An interesting property of rolling objects is seen when the 3.5″ and 5″ solid discs travel down the inclined plane equally and reach the bottom at the same time. This happens because objects of the same mass distribution (density) and shape have the same "resistance to mass" ratio. This means they all resist a change in motion equally, *regardless* of their mass or their size. The actual moment of inertia will be larger for a larger, more massive solid disc than for a smaller solid disc, but the "resistance" (a combination of the mass and the relative moment of inertia) relative to the mass will be the same for both solid discs. The "resistance to mass" ratio is larger for a ring than for a solid disc, and therefore the ring will always lose the race down the inclined plane to the solid disc, no matter what its size (theoretically). Please read further for a more technical (mathematical) explanation of the *Ring and Discs*.

A more advanced approach to describe the *Ring and Discs* demonstration incorporates kinetic and potential energy, and a further discussion of the moment of inertia. (Torque and angular acceleration can also be used to explain the ring and disc. Please refer to the references at the end of this activity for more information about these topics.) When an object is at the top of the inclined plane, it has potential energy (stored energy). Potential energy (PE) is equal to the weight of the object, which equals the mass (m) times the acceleration from gravity (g), times the relative height (h) of the object (see Equation 1 and Figure 2).

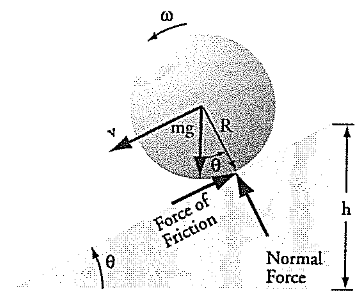


Figure 2

$$PE = mgh \qquad \textit{Equation 1}$$

As the object begins to move down the inclined plane, the potential energy is converted into kinetic energy (energy of motion). For a rolling object, the motion is both linear (straight down the inclined plane) and rotational (the object rolls about its central axis), so two forms of kinetic energy are involved. Linear kinetic energy (KE_l) is related to the mass (m) and linear speed (v) of the object (Equation 2). Rotational kinetic energy (KE_r) is related to the moment of inertia (I) of the rolling object about the rotational axis and the rotational speed (ω; the Greek letter *omega*) of the rolling object (Equation 3). (Notice the similarity between Equation 2 and Equation 3.) So, the total kinetic energy (KE_T) of a rolling object is equal to the linear kinetic energy plus the rotational kinetic energy (Equation 4).

$$KE_l = \tfrac{1}{2} mv^2 \qquad \textit{Equation 2}$$

$$KE_r = \tfrac{1}{2} I\omega^2 \qquad \textit{Equation 3}$$

$$KE_T = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 \qquad \textit{Equation 4}$$

In this demonstration, the ring and discs roll without slipping, that is, the point on the ring and disc in contact with the surface of the inclined plane is instantaneously at rest with respect to the inclined plane. This is due to the frictional force between the surface of the rolling object and the surface of the inclined plane acting against, and balancing, the force of gravity pulling the object down. Since there is no slipping across the two surfaces, energy will not be dissipated or lost as heat (it is a *conservative force*). Therefore, all the potential energy the rolling objects have when they are at the top of the inclined plane (before they begin to move) will be converted into kinetic energy at the bottom (Equation 5).

$$PE = KE_T \qquad \textit{Equation 5}$$

Or, for a rolling object:

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2 \qquad \textit{Equation 6}$$

Equation 6 can now be used to determine the speed of rolling objects when they reach the bottom of the inclined plane. For rotational motion, the rotational speed is related to the linear speed by the radius (R) of the object (Equation 7).

$$\omega = v/R \qquad \textit{Equation 7}$$

Substituting Equation 7, into Equation 6:

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I(v/R)^2 \qquad \textit{Equation 8}$$

Next, solve Equation 8 for v^2:

$$mgh = \tfrac{1}{2} v^2[m + I\,(1/R)^2] \qquad \textit{Equation 9}$$

$$v^2 = 2\,mgh/[m + I\,(1/R)^2] \qquad \textit{Equation 10}$$

Equation 10 represents the speed of a rolling object at the bottom of the inclined plane. The object that will have the highest speed at the bottom of the inclined plane will be the first to reach the bottom. The denominator $[m + I\,(1/R)^2]$ represents the "resistance" (total inertia) of the object, which was mentioned earlier.

What factors determine the speed of the rolling object at the bottom of the inclined plane? Using Equation 10, and the moment of inertia for a solid disc and a ring, the speed at the bottom of the inclined plane for the ring and disc can be calculated and compared. The moment of inertia (I), as discussed earlier, is dependent on the mass and the distribution of mass about an axis of rotation. The general form is:

$$I_{axis} = \Sigma\, m_i r_i^2$$

I_{axis} = moment of inertia about a particular axis of rotation

Σ = summation over all the infinitely small masses in the object

m_i = an infinitely small point mass in the object

r_i = the perpendicular distance between axis of rotation and m_i

Calculus (another credit to Isaac Newton) can be used to calculate the moment of inertia of many objects, but that is beyond the scope of this discussion (please refer to the references for more information on the derivation of the moment of inertia for different objects). The Moment of Inertia Table at the end of this discussion has the moment of inertia equations for some familiar objects. From the table, the moment of inertia for a solid disc is $I_{solid\ disc} = \frac{1}{2} mR^2$, and the moment of inertia for a ring is $I_{ring} = mR^2$. Substituting these expressions into Equation 10 to solve for the speed at the bottom of the inclined plane for the solid disc and ring, respectively:

$$v^2_{solid\ disc} = 2\ mgh/[(m + (\tfrac{1}{2} mR^2)(1/R)^2] = 2\ mgh/(m + \tfrac{1}{2} m) = 2\ mgh/(\tfrac{3}{2} m)$$

$$v^2_{solid\ disc} = 4/3\ gh \qquad \textit{Equation 11}$$

$$v^2_{ring} = 2\ mgh/[(m + (mR^2)(1/R^2)] = 2\ mgh/(m + m) = 2\ mgh/(2m)$$

$$v^2_{ring} = gh \qquad \textit{Equation 12}$$

Equation 11 and Equation 12 show that the solid disc will be moving faster at the bottom of the inclined plane than the ring, so it will take less time for the solid disc to reach the bottom. They also show that the speed is independent of the mass or the size of the disc or ring. The solid disc will always win. The only factor influencing the speed of descent is the shape of the rolling object. Refer to the Moment of Inertia Table to determine which object will win the race, a solid ball or a solid disc?

Connecting to the National Standards

This laboratory activity relates to the following National Science Education Standards (1996):

Unifying Concepts and Processes: Grades K–12
Evidence, models, and explanation

Content Standards: Grades 5–8
Content Standard B: Physical Science, understanding of motions and forces, transfer of energy

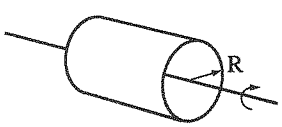
Content Standards: Grades 9–12
Content Standard B: Physical Science, motions and forces, interactions of energy and matter

References

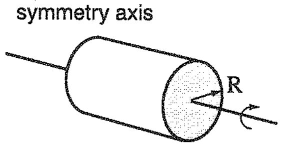
Hewitt, Paul G. *Conceptual Physics,* 3rd Ed.; Addison Wesley: Menlo Park, California, 1999; pp 157–158.

Tipler, Paul A. *Physics for Scientists and Engineers,* 3rd Ed., Vol. 1; Worth Publishers: New York, 1990; pp 231–239 and 249–254.

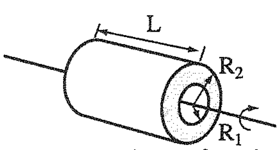
Moment of Inertia Table



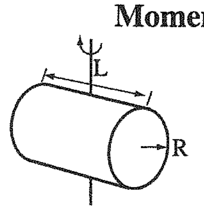
$I = mR^2$
Cylindrical shell about the symmetry axis



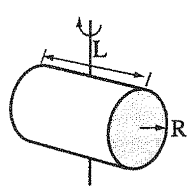
$I = \frac{1}{2} mR^2$
Solid cylinder about the symmetry axis



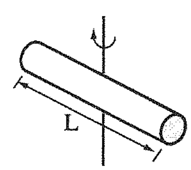
$I = \frac{1}{2} m(R_1^2 + R_2^2)$
Hollow cylinder about the symmetry axis



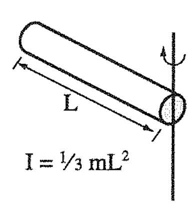
$I = \frac{1}{2} mR^2 + \frac{1}{12} mL^2$
Cylindrical shell about the diameter through center



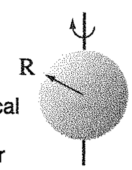
$I = \frac{1}{4} mR^2 + \frac{1}{12} mL^2$
Solid cylinder about the diameter through center



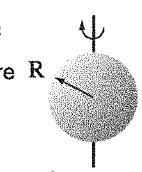
$I = \frac{1}{12} mL^2$
Thin rod about a perpendicular axis through center



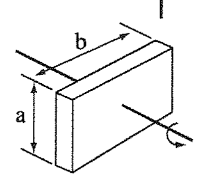
$I = \frac{1}{3} mL^2$
Thin rod about a perpendicular axis through one end

$I = \frac{2}{3} mR^2$


Thin spherical shell about the diameter

$I = \frac{2}{5} mR^2$


Solid sphere about the diameter



$I = \frac{1}{12} m(a^2 + b^2)$
Solid rectangular parallel-piped about the axis, perpendicular to face, through center

The *Ring and Discs—Moment of Inertia Demonstration Kit* is available from Flinn Scientific, Inc.

Catalog No.	Description
AP4634	Ring and Discs

Consult your *Flinn Scientific Catalog/Reference Manual* for current prices.